Verb Phrase Ellipsis using Frobenius Algebras in Categorical Compositional Distributional Semantics

Dimitri Kartsaklis   **Matthew Purver**   **Mehrnoosh Sadrzadeh**
School of Electronic Engineering and Computer Science
Queen Mary University of London
Mile End Road, London E1 4NS, UK
{d.kartsaklis, m.purver, mehrnoosh.sadrzadeh}@qmul.ac.uk

Abstract

We sketch the basis of a categorical compositional distributional semantic approach to the analysis of verb phrase ellipsis.

1 Introduction: Verb Phrase Ellipsis

The term *ellipsis* covers a range of pragmatic phenomena in which parts of the semantic content of an expression are not represented explicitly in its lexical form, but can be recovered implicitly from meaning established in context. In the category of *verb phrase ellipsis (VPE)*, a light verb or auxiliary is used to stand in for the meaning of (part of) a verb phrase established in context, with this ellipsis site often marked by discourse markers or coordinators such as *"too"*, *"so"*, *"and"*. The antecedent context may be within the same sentence (or utterance) or in a previous one. Examples (1a-c) show a range of basic possibilities: in each case, the full content conveyed includes the proposition that Sandy wears (or might wear) a hat.

(1) a. Kim wears a hat, and so does Sandy.

　　 b. Kim wears a hat. Sandy does too.

　　 c. Kim wears a hat. Sandy might.

Conventional Approaches   Formal approaches to the analysis of VPE must therefore explain how this full content is obtained. Existing approaches have generally taken either a syntactic or a semantic approach. Syntactic approaches assume that ellipsis involves deletion of syntactic structure at the ellipsis site (provided that it matches structure available from the antecedent), and/or reconstruction by copying across antecedent structure (see e.g. (Fiengo and May, 1994; Merchant, 2004)). However, these run into problems when explaining the multiple interpretations available in examples like (2), where on one (*"strict"*) reading Sandy wears Kim's hat and on another (*"sloppy"*) reading, Sandy wears Sandy's hat (and similar ambiguity of quantification arises in (1)).

(2) Kim wears his hat. Sandy does too.

Semantic approaches, e.g. (Dalrymple et al., 1991; Pulman, 1997), solve this problem by assuming that ellipsis involves underspecification of content, to be resolved by producing a predicate via a suitable abstraction from the antecedent. If (3)a shows a semantic logical form representing (1)a, resolution corresponds to the steps in (3)b-c, with 3b giving the strict and 3c the sloppy reading:

(3) a. $wear(k, hat(k)) \wedge P(s)$

　　 b. $P = \lambda x.wear(x, hat(k))$
　　　 $\rightsquigarrow wear(k, hat(k)) \wedge wear(s, hat(k))$

　　 c. $P = \lambda x.wear(x, hat(x))$
　　　 $\rightsquigarrow wear(k, hat(k)) \wedge wear(s, hat(s))$

Mixed syntactic/semantic approaches have also been proposed to cover wider ranges of phenomena; see e.g. (Kempson et al., 2015) for an overview. However, we are not aware of any work attempting to join ellipsis analysis with distributional or vector-space-based models of semantics. Here, we sketch how such an approach might look.

## 2 Categorical Compositional Distributional Semantics

Distributional models of meaning represent words by their collocation frequencies in corpora of text. They provide a model of meaning orthogonal to compositional models where grammatical structures are represented by functions over vacuous predicates. For instance, the lambda calculus model presented in the previous section provides a logical form for the compositional nature of an ellipsis, but says nothing about the content of the predicates 'wear' and 'hat'; these are

rather copied from the natural language to the logical language. Categorical compositional distributional semantics (Coecke et al., 2010; Grefenstette and Sadrzadeh, 2015) was developed to combine the distributional models of meaning with the compositional ones. The apparent goal is to become able to compose the distributional vectors of words and obtain vector representations for phrases and sentences. The ultimate goal of our project is to extend that setting to dialogue and discourse, a part of whose constructions rely on different forms of ellipsis. In this paper we deal with VP ellipsis, other forms can be dealt with using similar constructions.

These models consist of compact closed categories, which have objects, their left and right adjoints, morphisms and tensors. When reasoning about vectors, the objects become vector spaces, the elements within them vectors. These are depicted using triangles with legs emanating from them, each leg representing a tensor:
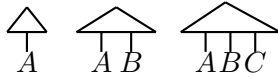

One can compose the vectors and tensors with each other and reason about a flow of information amongst them using a cap-cup wiring:
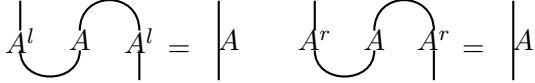
One can copy and merge some of this information, by assuming that the objects corresponding to them have a *Frobenius Algebra* over them. Each Frobenius algebra is of a pair of an algebra and a coalgebra, amounting to extra wirings as follows:

In order for information to flow between these objects, they do not have to correspond to adjacent words. Frobenius algebras and cap-cup wirings have been used in previous work to reason about clauses whose meanings rely on a notion of movement; in our terms, a flow of information (Sadrzadeh et al., 2013; Sadrzadeh et al., 2016). For instance, the meaning of an object relative clause 'man that dog chased' is obtained by passing (or flowing) the information of 'man' to 'chased':
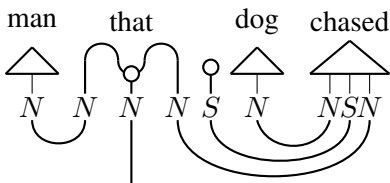

In the above, 'that' has a Frobenius structure, which is used in to combine the information in 'man' with that of 'dog chased' and return the meaning of the phrase. Linear algebraically, the above diagram is equivalent to

$$\overrightarrow{man} \odot (\overrightarrow{chased} \times \overrightarrow{dogs})$$

A similar mechanism has been used in (Kartsaklis, 2015) to reason about coordinators, which we will use to model ellipsis.

3 Approach

Consider the sentence "John sleeps and Bill does too"; the handling of the ellipsis is hard-wired in the structure of to the coordinator, in the following way:
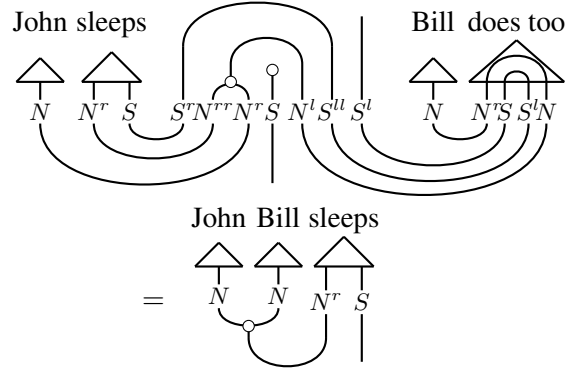
In the above diagram, the segment 'does too' acts as an identity tensor allowing the interaction of the verb 'sleeps' with the second subject ('Bill'); in inter-sentential variants such as (1)b,c this tensor could be associated with a discourse relation (see e.g. (Asher and Lascarides, 2003)). Within the coordinator tensor, the two subjects ('John' and 'Bill') are merged together with Frobenius multiplication and interact as one with the verb of the sentence, as it is evident by the normal form at the lower part of the diagram. Linear-algebraically, this becomes:

$$(\overrightarrow{John} \odot \overrightarrow{Bill})^{\top} \times \overrightarrow{sleeps}$$

Specific interpretations of this can give strict and sloppy readings (Abramsky and Sadrzadeh, 2014). The morphism of the coordinator is the following:

$$(1_s \otimes \Delta_N \otimes \iota_S \otimes 1_{N \otimes S \otimes S}) \circ (1_S \otimes \eta_N \otimes 1_{S \otimes S}) \circ (\eta_S \otimes 1_S)$$

Here, the δ_N is the Frobenius combining map and η and ϵ are the co-unit and unit of compact closure and correspond to the cap and cup of diagrams.

References

Samson Abramsky and Mehrnoosh Sadrzadeh. 2014. Semantic unification: a sheaf theoretic approach to natural language. In Casadio et al, editor, *Categories and Types in Logic, Language, and Physics, Essays dedicated to Jim Lambek on the occasion of his 90th birthday*, number 8222 in LNCS, pages 1–13.

Nicholas Asher and Alex Lascarides. 2003. *Logics of Conversation*. Cambridge University Press.

B. Coecke, M. Sadrzadeh, and S. Clark. 2010. Mathematical Foundations for Distributed Compositional Model of Meaning. Lambek Festschrift. *Linguistic Analysis*, 36:345–384.

Mary Dalrymple, Stuart M. Shieber, and Fernando C. N. Pereira. 1991. Ellipsis and higher-order unification. *Linguistics and Philosophy*, 14(4):399–452.

Robert Fiengo and Robert May. 1994. *Indices and Identity*. MIT Press, Cambridge, Mass.

Edward Grefenstette and Mehrnoosh Sadrzadeh. 2015. Concrete models and empirical evaluations for the categorical compositional distributional model of meaning. *Computational Linguistics*, 41:71–118.

Dimitri Kartsaklis. 2015. *Compositional Distributional Semantics with Compact Closed Categories and Frobenius Algebras*. Ph.D. thesis, University of Oxford.

Ruth Kempson, Ronnie Cann, Arash Eshghi, Eleni Gregoromichelaki, and Matthew Purver. 2015. Ellipsis. In S. Lappin and C. Fox, editors, *Handbook of Contemporary Semantic Theory*, chapter 4. Wiley, 2nd edition, September.

J. Merchant. 2004. Fragments and ellipsis. *Linguistics and Philosophy*, 27:661–738.

Stephen G. Pulman. 1997. Higher order unification and the interpretation of focus. *Linguistics and Philosophy*, 20(1):73–115.

M. Sadrzadeh, Clark S., and C. Coecke. 2013. Frobenius anatomy of word meanings i: subject and object relative pronouns. *Journal of Logic and Computation*, 23:1293–1317.

M. Sadrzadeh, Clark S., and C. Coecke. 2016. Frobenius anatomy of word meanings 2: possessive relative pronouns. *Journal of Logic and Computation*, 26:785–815.