CO-HISTOGRAM AND ITS APPLICATION IN REMOTE SENSING IMAGE COMPRESSION EVALUATION*

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ABSTRACT

Peak signal-to-noise ratio (PSNR) has found its application as an evaluation metric for image coding, but in many instances it provides an inaccurate representation of the image quality. The new tool proposed in this paper is called co-histogram, which is a statistic graph generated by counting the corresponding pixel pairs of two images. For image coding evaluation, the two images are the original image and a compressed and recovered image. The graph is a two-dimensional joint probability distribution of the two images. A co-histogram shows how the pixels are distributed among combinations of two image pixel values. By means of co-histogram, we can have a visual interpretation of PSNR, and the symmetry of a co-histogram is also significant for objective evaluation of remote sensing image compression. Our experiments with two SAR images and a TM image using DCT-based JPEG and wavelet-based SPIHT coding methods perform the importance of the co-histogram symmetry.

1. INTRODUCTION

Remote sensing images are of huge data size and with abundant minutiae, so it is necessary but difficult to compress. More and more image coding methods are proposed while none of them are confirmed to be the best. The subjective evaluation methods are human vision oriented. The subjective metrics suffer from the physiological, psychological and environmental impact on the viewers. Therefore, the subjective metrics are not appropriate to evaluate the remote sensing images that are mainly computed and interpreted by computers. Great effort has been made to assess image quality objectively but close to the subjective evaluation, such as the fuzzy image metric close to the subjective mean opinion score [2], information mean square error using a local information map to weight the squared error [4]. However, image compression evaluation is so difficult that only limited success has been achieved.

So far, the widely used objective metrics for compressed image quality assessment are peak signal-to-noise ratio (PSNR) and mean square error (MSE). If the peak value of an image (the maximum minus the minimum) is 255, the mathematical representations are

$$PSNR = 10\log_{10}\frac{255^2}{MSE}$$

$$MSE = \frac{1}{MN}\sum_{x=1}^{M}\sum_{y=1}^{N}(f(x,y)-g(x,y))^2$$

where $f(x,y)$ and $g(x,y)$ are two images of size MxN.

However, they are considered not so satisfying. In many instances, it provides an inaccurate representation of the image quality.

An ideal objective metric should be reliable, easily-computable and directly-intercomparable. There are two ways to overcome the shortcomings of PSNR: (1) to find another better objective metric to replace PSNR and to assess image coding more effectively, and (2) to find another objective metric as a complement to reinforce PSNR and to jointly assess image coding more comprehensively. This paper is mainly based on the latter idea.

The new tool proposed in this paper is called co-histogram. A co-histogram is a statistic graph generated by counting the corresponding pixel pairs of two images. A co-histogram is also a two-dimensional joint probability distribution of the two images. A co-histogram shows how the pixels are distributed among combinations of two image pixel values, and visually it also gives an intuitive interpretation of PSNR.

For remote sensing image compression evaluation, a co-histogram can be easily obtained and then the metrics, the corresponding PSNR and its symmetry, are easily computable. Our experiments with two SAR images and a TM image using DCT-based JPEG and wavelet-based SPIHT coding methods perform the reliability of the co-histogram symmetry for image coding assessment.

* This work was supported by NKBRSF China under Grant G1998030606 and the funding for the recipients of the National Excellent Doctoral Dissertation, China, under Grant 200038.

2. COMPRESSION OF REMOTE SENSING IMAGES

A remote sensing image possesses two spatial dimensions and maybe one more component or spectral dimension. Those images with the third dimension are called multi-spectral images, and hyper-spectral images are named for those with high spectral resolution.

A generic image compression model is illustrated in Figure 1.

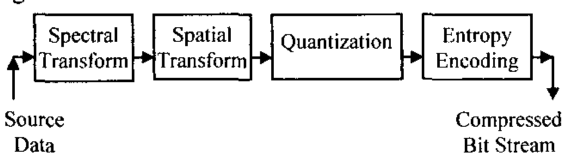


Figure 1. A generic image coding model

From the entropy principles, we know that the lossless compression ratio is very limited notwithstanding a lossless encoding method results in a complete recovery of the data without any change or loss. In contrast, the lossy compression ratio can be very high and yet higher compression ratio reflects greater loss of the data. In the case that a data loss is inevitable or acceptable, for remote sensing image compression, we just expect that (1) the noise in an image could be removed; (2) all the important information should be kept; (3) in the applications such as image classification, the amounts misclassified into and that out of a class should be statistically as close as possible so that the final classification error can be reduced much.

Among above expectations, noise elimination leads to smoothness, which is a common consequence of lossy image compression techniques. Nevertheless, the noise of either Gaussian distribution or uniform distribution is symmetrical about the signal average. For our second expectation, information consists in changes of data, and image information takes shape at image edges. To determine what edges are useful depends on the applications. For instance, in the military applications of remote sensing imagery, the small features of military objects are significant, while the larger scale edges are more important in the earth resource and environmental applications. The latter applications also expect better classification accuracy. The problem is if we can estimate the classification accuracy without classification. It is the main point that our paper originates from.

3. CO-HISTOGRAM AND ITS PROPERTIES

A histogram of an image is a distribution statistics of the image pixel values. It finds many applications such as image enhancement, thresholding, retrieval, classification and recognition.

The probability that pixel value p occurs in a digital image $f(x,y)$ of size M-by-N, $H(p)$, is counted as

$$H(p)=\frac{1}{MN}\sum_{y=1}^{M}\sum_{x=1}^{N}\delta(f(x,y),p)$$

where $\delta(f,p)$ is the Kronecker function:

$$\delta(f,p)=\begin{cases}1 & (f=p)\\ 0 & (f\neq p)\end{cases}$$

For all possible p, $H(p)$ makes the histogram of image $f(x,y)$.

For two images of the same size MxN, $f(x,y)$ and $g(x,y)$, the joint probability that pixel value pair (p,q) occurs is

$$H(p,q)=\frac{1}{MN}\sum_{y=1}^{M}\sum_{x=1}^{N}\delta(f(x,y),p)\cdot\delta(g(x,y),q)$$

For all possible pixel value pair (p,q), $H(p,q)$ makes the co-histogram of image pair $f(x,y)$ and $g(x,y)$.

A co-histogram has following properties.

(1) Axial projection

$$\sum_{q}H(p,q)=\frac{1}{MN}\sum_{q}\sum_{y=1}^{M}\sum_{x=1}^{N}\delta(f(x,y),p)\cdot\delta(g(x,y),q)=H(p)$$

(Histogram of image $f(x,y)$)

$$\sum_{p}H(p,q)=\frac{1}{MN}\sum_{p}\sum_{y=1}^{M}\sum_{x=1}^{N}\delta(f(x,y),p)\cdot\delta(g(x,y),q)=H(q)$$

(Histogram of image $g(x,y)$)

(2) Mean pixel value

$$I_f=\sum_{p}pH(p)=\sum_{p,q}pH(p,q)$$

$$I_g=\sum_{q}qH(q)=\sum_{p,q}qH(p,q)$$

If $I_f=I_g$, we have

$$\sum_{p,q}(p-q)H(p,q)=0$$

(3) Variance of pixel values

$$Var(f)=\sum_{p}(p-I_f)^2H(p)=\sum_{p,q}p^2H(p,q)-I_f^{\,2}$$

$$Var(g)=\sum_{q}(q-I_g)^2H(q)=\sum_{p,q}q^2H(p,q)-I_g^{\,2}$$

If $I_f=I_g$, we have

$$Var(f)-Var(g)=\sum_{p,q}(p^2-q^2)H(p,q)$$

(4) Diagonal projection

Projection along the line $p=q$, or its parametric representation, $p(t)=t+r$ and $q(t)=t$,

$$\begin{aligned}\sum_{t}H(p(t),q(t))&=\sum_{t}H(t+r,t)\\&=\sum_{x,y}\sum_{t}\delta(f(x,y),t+r)\cdot\delta(g(x,y),t)\\&=\sum_{x,y}\delta(f(x,y)-g(x,y),r)\\&=H(r)\end{aligned}$$

In fact, it is the histogram of the difference image:

$r(x,y) = f(x,y) - g(x,y)$.

(5) Mean of the difference image

$$I_r = \sum_r rH(r) = \sum_r r\sum_t H(t+r,t) = \sum_{p,q}(p-q)H(p,q) = I_f - I_g$$

(6) Variance of the difference image

$$\begin{aligned} Var(r) &= \sum_r (r-I_r)^2 H(r) = \sum_r r^2 H(r) - I_r^2 \\ &= \sum_r \sum_t r^2 H(t+r,t) - I_r^2 \\ &= \sum_{p,q}(p-q)^2 H(p,q) - (I_f - I_g)^2 \\ &= MSE - (I_f - I_g)^2 \end{aligned}$$

where MSE is the mean square error between two images.

For image compression, we take the original image and a compressed and recovered image as above two images $f(x,y)$ and $g(x,y)$. Thus, for most compression techniques, we generally have $I_f \approx I_g$. Accordingly, the image coding evaluation metrics PSNR and MSE correspond to the variance of the difference image, and then to the variance of the diagonal projection of the co-histogram, which is visually a "width" of the co-histogram. Therefore, PSNR is a measure of the co-histogram width along the diagonal.

If $Var(r) = 0$, or the whole co-histogram is strictly on the diagonal, the corresponding image compression must be lossless. If $Var(r) \neq 0$, or the off-diagonal distribution is not entirely zero, the employed image compression method must be a lossy one. Thus, the PSNR can be calculated directly from the co-histogram:

$$\begin{aligned} PSNR &= 10\log_{10}\frac{255^2}{MSE} \\ &= 10\log_{10}255^2 - 10\log_{10}MSE \\ &\approx 48.1308 - 10\log_{10}(Var(r)) \end{aligned}$$

4. SYMMETRY OF CO-HISTOGRAM

An important technique for remote sensing image applications is classification statistics. Of course, the closer the amounts misclassified into and out of a class are, the higher the classification accuracy is, and the better the compression method should be.

The current classification techniques, supervised or unsupervised, are principally based on the pixel values, so the classification co-histogram is highly related to the image co-histogram. In the classical paper by Haralick et al [1], it is mentioned that the classification accuracy is 74-77% if the pixel values are used only. Therefore, the classification co-histogram principally relates to the image co-histogram, of which the symmetry is a significant metric for classification accuracy estimation.

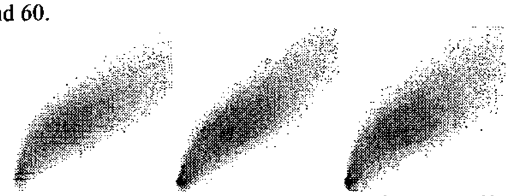
On the other side, the symmetry of a co-histogram is mathematically independent of the PSNR, and therefore is a valid complement to PSNR. Symmetry together with PSNR makes perfect the depiction of co-histogram.

The symmetry of a co-histogram is defined as:

$$Symmetry = \frac{\sum_{p\geq q} H(p,q)H(q,p)}{\sqrt{\sum_{p\geq q} H^2(p,q)\sum_{p\geq q} H^2(q,p)}}$$

5. EXPERIMENTS AND ANALYSIS

The images used to illustrate the validity and effectiveness of co-histogram are two SAR images (image A and B, 1 band, 1024x1024) and a TM satellite image (6 bands, 512x512). The compression methods we used are DCT-based JPEG [5] and wavelet-based SPIHT [3]. The classification method we used is a supervised method. The co-histograms in this paper are represented by images, in which the gray levels stand for the joint probability of pixel pairs, the horizontal axis is for the pixel values in the original image, the vertical axis is for the pixel values in the compressed image, and both axes range the same as the image pixel values, from 0 to 255. Figure 2 gives some co-histograms of a SAR image compressed with JPEG at compression ratio of 30 and with SPIHT at 30 and 60.



(a) JPEG at 30 (b) SPIHT at 30 (c) SPIHT at 60

Figure 2. Co-histograms of SAR image B and its compressed versions

With the original image, the classified categories of SAR image A are water 53.74%, soil 44.36% and vegetation 1.90%, and the percentages of SAR image B are 22.87%, 74.82% and 2.31%. Compared with compressed images, image quality metric PSNR and our proposed symmetry are listed in Table 1 and the classification error and the symmetry of classification co-histogram are listed in Table 2.

In Table 1, PSNR of compression with JPEG at ratio about 30 is close to that with SPIHT at 60, but the former co-histogram symmetry is lower than that of the latter and that with SPIHT at 30.

In Table 2, the classification error of compression with JPEG at ratio 30 is obviously less than that with SPIHT at 30 and 60. In comparison with Table 1, it gives evidence of the importance of co-histogram symmetry. From the co-histogram symmetry of images and classification images, we can further conclude that classification raises the co-histogram symmetry but doesn't change the relative order.

The land use classification of the original TM image gives 7 categories: water 6.91%, trees/crops 20.31%, corn 45.43%, inhabited 12.77%, newly-cropped 6.80%, orchard 3.37% and barren land 4.41%. The image is compressed with JPEG at five various quality options, 100%, 75%, 50%, 25% and 10%. The corresponding compression ratios are 1.60, 10.25, 17.38, 30.56 and 55.27. Seven compression ratios with SPIHT are 2, 8, 10, 15, 25, 32 and 50.

Figure 3 of compression ratio (CR) vs. PSNR illustrates that SPIHT is better than JPEG. Figure 4 of compression ratio vs. co-histogram symmetry (CHS) shows that SPIHT is better than JPEG as well, especially at large compression ratios. Figure 5 of compression ratio vs. classification co-histogram symmetry (CCHS) agrees with Figure 4, monotone decreasing and the value relativity hold. Figure 6 of compression ratio vs. classification accuracy (CA) shows the classification accuracy with SPIHT is always higher than that with JPEG.

From above figures for the TM data and those tables with SAR images, we know the wavelet-based SPIHT method has advantage over the DCT-based JPEG method. Although the relationship between co-histogram symmetry and classification accuracy for the TM data is not so marvelous as that for SAR images, the symmetry relativity doesn't change from the image co-histograms to the classification co-histograms.

6. CONCLUSIONS

(1) Co-histogram provides a visual interpretation of PSNR.
(2) Co-histogram symmetry is a valid complement to PSNR and is also an objective statistic metric. Symmetry together with PSNR makes perfect the depiction of co-histogram. Its calculation is direct and as easy as PSNR.
(3) Co-histogram symmetry is significant in evaluation of remote sensing image compression.
(4) A byproduct of this paper: the wavelet-based SPIHT method is better for remote sensing image compression than the DCT-based JPEG. Methods like SPIHT that give encoding priority according to the data significance perform better and serve better in image applications.
(5) Co-histogram merits further investigation into other image applications.

7. REFERENCES

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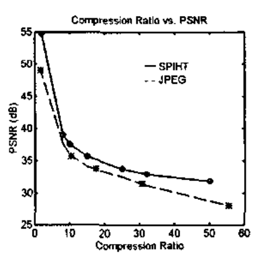
Table 1. PSNR and image co-histogram symmetry

Compression Ratio (Method)	SAR image A		SAR image B	
	PSNR	Symmetry	PSNR	Symmetry
JPEG-30	23.83	0.2442	21.65	0.1898
SPIHT-30	25.37	0.8256	22.70	0.8036
SPIHT-60	23.87	0.8042	21.49	0.7923

Table 2. Classification error and classification co-histogram symmetry (SAR images)

Compress Method-Ratio	SAR image A				SAR image B			
	Classification Error			Symm	Classification Error			Symm
	Water	Soil	Veget.		Water	Soil	Veget.	
JPEG-30	4.70	5.57	0.87	.9973	6.78	8.10	1.32	.9959
SPIHT-30	2.59	2.90	0.32	.9992	4.39	5.06	0.67	.9983
SPIHT-60	3.18	3.68	0.50	.9988	5.20	6.18	0.98	.9975



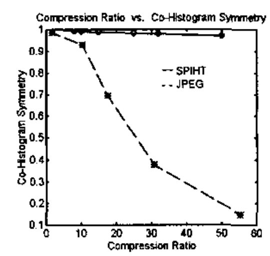
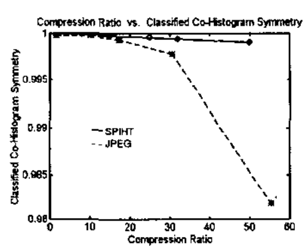


Figure 3 CR vs. PSNR (TM) **Figure 4** CR vs. CHS (TM)



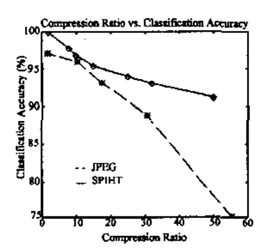


Figure 5. CR vs. CCHS (TM) **Figure 6.** CR vs. CA (TM)